Calculations Policy

(in line with the requirements of the 2014 Primary Mathematics National Curriculum)

Introduction

The 2014 Primary National Curriculum for Mathematics differs from its predecessor in many ways. Alongside the end of year expectations, there is also an emphasis on depth before breadth and a greater expectation of what children should achieve.

One of the key differences is the level of detail included, indicating what children should be learning and when. This is suggested content for each year group, but schools have been given autonomy to introduce content earlier or later, with the expectation that by the end of each key stage the required content has been covered. This document sets out progression in the teaching of written calculation strategies; it is up to individual schools to decide at what stage these strategies should be introduced. To support this process, progression in the requirements of the 2014 National Curriculum is recorded alongside.

Purpose

This policy makes teachers aware of the written strategies that children are formally taught as they progress through primary school. The policy only details the strategies – teachers must plan opportunities for children to apply these; for example, when solving problems, developing reasoning skills or where opportunities emerge elsewhere in the curriculum. Supporting understanding through the use of pictorial representations and concrete materials is a key factor in children understanding the strategies they are being taught.

Mathematical language

The 2014 National Curriculum is explicit in articulating the importance of children using the correct mathematical language as a central part of their learning. Indeed, in certain year groups, the non-statutory guidance highlights the requirement for children to extend their language around certain concepts. It is therefore essential that teaching using the strategies outlined in this policy is accompanied by the use of appropriate mathematical vocabulary. New vocabulary should be introduced in a suitable context and explained carefully. High expectations of the mathematical language used are essential, with teachers only accepting what is correct. For example, using the term 'regroup' rather than 'carry' or 'borrow.'

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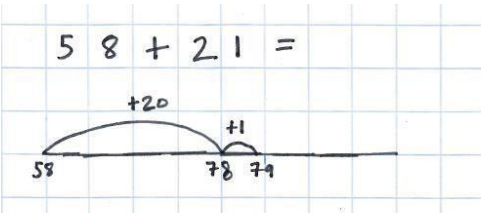
ADDITION

2014 National Curriculum: Addition		
Year 3: Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction	**Year 4:** Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate	**Year 5:** Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)

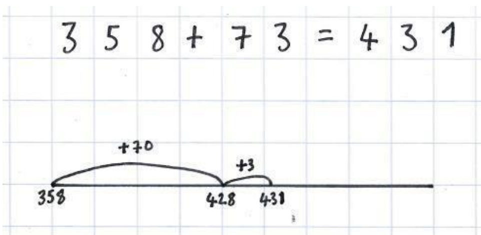
1. Partitioning into tens and ones using a number

- TO + TO (up to 100)



Adding the tens followed by the ones. Taught with Dienes or place value counters alongside.

- HTO + TO (through 100)



Moving through 100 adding the tens followed by the ones. Taught with Dienes or place value counters alongside.

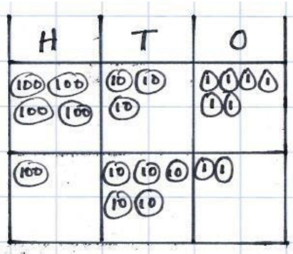
2. Vertical Partitioning

85 + 37 = ?
85 + 30 = 115
115 + 7 = 122

Introducing the vertical method by adding the tens then the ones.

3. Expanded Column Method

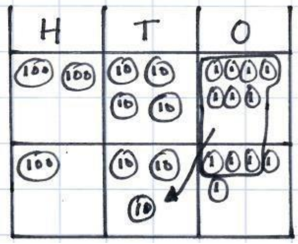
- Use place value counters or Dienes



400 + 30 + 6
100 + 50 + 2
500 + 80 + 8

Introduction to the column method through partitioning. This should be introduced alongside the concrete or pictorial representation.

- Leading to regrouping (HTO + TO moving onto HTO + HTO)



247 + 125 =

200	+	40	+	7			
100	+	20	+	5			
300		70		2	=	372	
		10					

Regrouping demonstrates how, for example, twelve ones is the same as one ten and two ones. This should be introduced alongside the concrete or pictorial representation.

4. Introduction to Formal Method (expanded)

	7	5	7	
+	1	2	1	
			8	(7 + 1)
		7	0	(50 + 20)
	8	0	0	(700 + 100)
	8	7	8	

Again, the method is expanded to aid understanding. Children should be adding from the right – ie starting with the ones, then the tens and finally the hundreds. Initially, equations for ones, tens and hundreds can be written in brackets alongside.

5. Formal Written Method (without regrouping and moving up to and beyond 4 digits

	6	3	2
+	1	5	4
	7	8	6

The formal written method without any expanding is introduced for the first time. Teachers may want to include H, T and O at the top of columns to support.

6. Formal Written Method (with regrouping and moving up to and beyond 4 digits)

	6	4	7	3
+	2	4	6	1
	8	9	3	4
		1		

The formal written method is extended to include regrouping. The regrouping should be recorded below the equals sign.

7. Formal Written Method involving decimals

£64.50 – £19.63 =

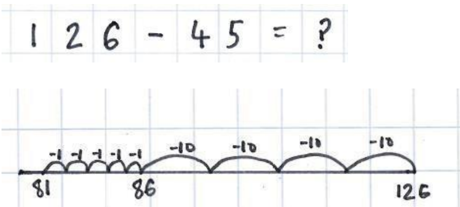
	£	6	4.	5	0
+	£	1	9.	6	3
	£	8	4.	1	3
		1	1		

The formal written method is extended to include decimals. Again, the regrouping should be recorded below the equals sign.

SUBTRACTION

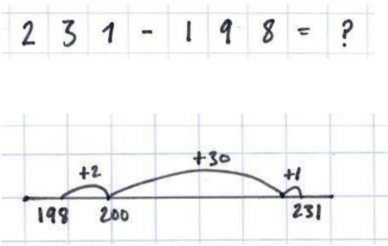
2014 National Curriculum: Subtraction		
Year 3: Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction	**Year 4:** Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate	**Year 5:** Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)

1. 'Counting back' on a number line



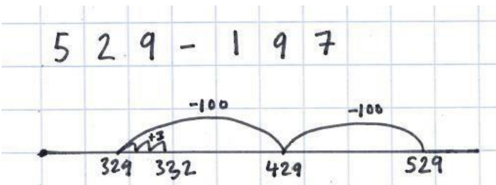
The jumps are recorded above the representation. Subtracting in tens before moving onto ones.

2. 'Counting on' on a number line



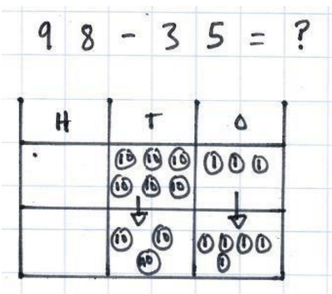
This method involves working out the difference between two numbers by counting on. The first jump should be to the next multiple of ten followed by counting in multiples of ten before adding any remaining ones. Another strategy would be to count in tens first and then in ones.

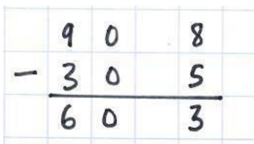
3. 'Counting back and adding ones' on a number line



This example shows counting back in 100s and then adding ones (to compensate). Other examples may be better suited to counting back in tens first then adding ones (to compensate).

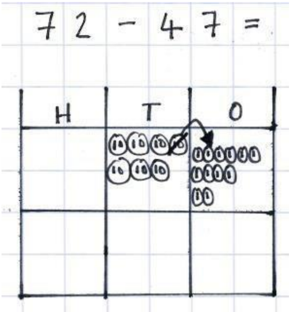
4. Expanded Column Subtraction (with regrouping and with place value counters or Dienes)





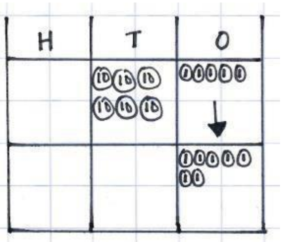
The pictorial and concrete representations demonstrate what is being taken away with the answer left behind. This is then recorded alongside in a vertical, expanded equation.

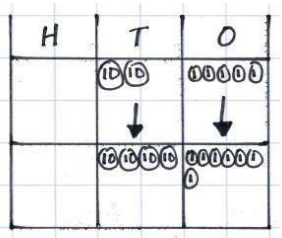
5. Expanded Column Subtraction (with regrouping and with place value counters or Dienes)

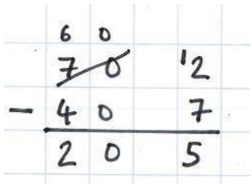


Where the ones cannot be subtracted, regrouping takes place. In this example, seven tens is regrouped into six tens and ten ones. The equation becomes 12-7 and 60-40 respectively.

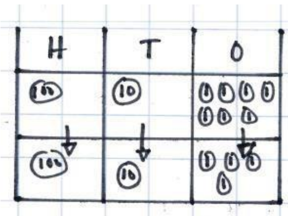
The three representations show what is happening with the equation recorded alongside at the end.

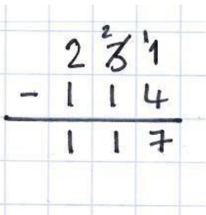






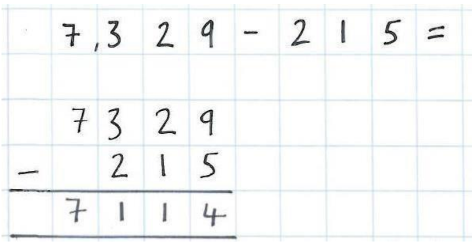
6. Formal Written Method (with regrouping and with place value counters or Dienes)





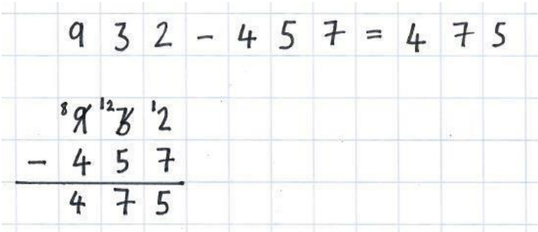
At this point, children are introduced to the formal written method without expanding for the first time. Again, in this example, the ones cannot be subtracted so regrouping takes place. Three tens are regrouped into two tens and ten ones.

7. Formal Written Method (without regrouping)



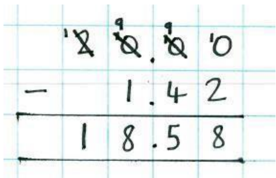
Children are now using the formal written method without concrete or pictorial support and are moving into thousands, without regrouping.

8. Formal Written Method (multiple regrouping)



Children are now using the formal written method without concrete or pictorial support and are required to regroup more than once.

9. Formal Written Method (with multiples of 100)

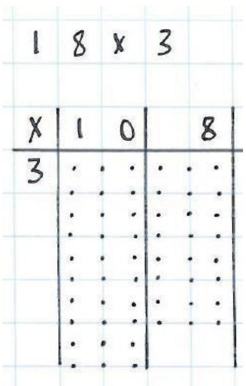


To complete subtraction, children are subtracting from multiples of 100 or, where there are multiple 0s, in the case of, for example, money. The multiple stages of regrouping may be difficult to represent.

MULTIPLICATION

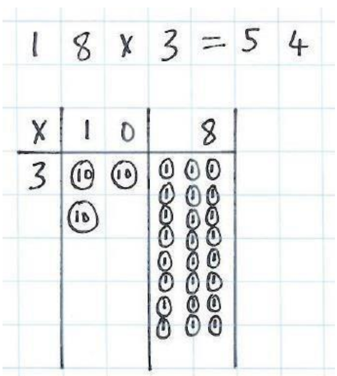
2014 National Curriculum: Multiplication			
Year 3: Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods	**Year 4:** Multiply two-digit and three-digit numbers by a one-digit number using formal written layout	**Year 5:** Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers	**Year 6:** Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication

1. Developing written methods using an array



The two digit number is partitioned horizontally with the tens digit coming first. The equation is then represented using counters (or an array).

2. Grid method (using place value counters or Dienes)

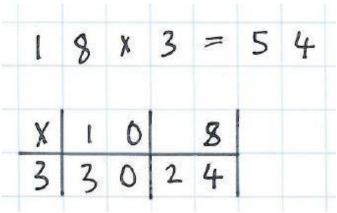


Again, the two digit number is partitioned horizontally with the tens digit coming first. This time the equation is represented using place value counters or Dienes.

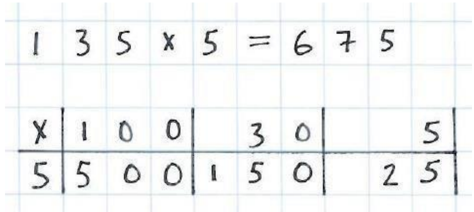
3. Grid method (using place value counters or Dienes)

- TO x O

The same layout is used as before but this time, the digits are being used.



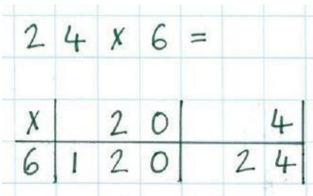
- HTO x O

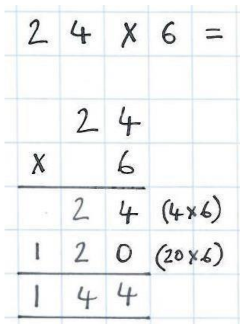


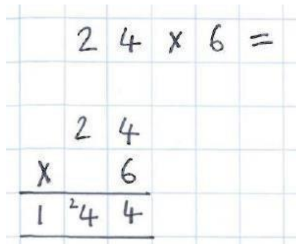
The three digit number is partitioned horizontally with the hundreds first followed by the tens and ones.

4. Short Multiplication (introduced alongside grid method)

- **TO x O**



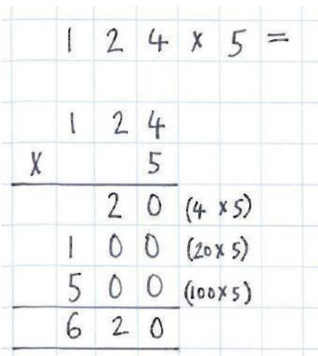


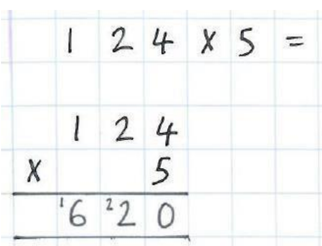


The short multiplication method is introduced alongside the grid method and expanded form to aid understanding.

Children should be encouraged to discuss what is similar and what is different between the different strategies.

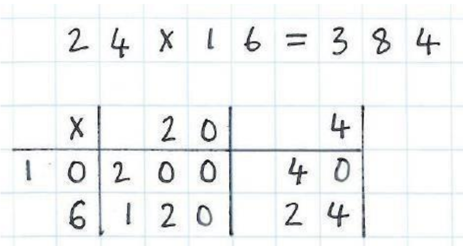
- HTO x O

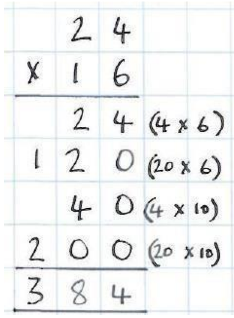


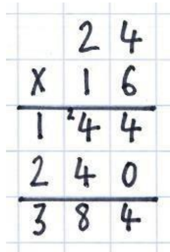


5. Long Multiplication (introduced alongside grid method)

- TO x TO



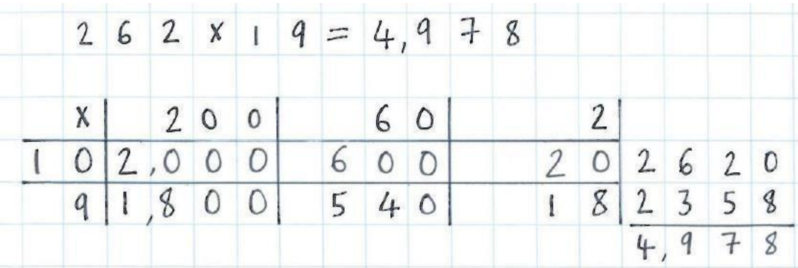


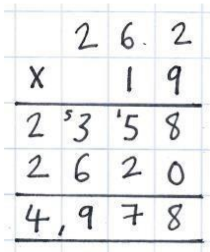


Multiplying a two digit number by a three digit number should be introduced through the grid method before moving to long multiplication to aid understanding.

When long multiplication is introduced, both equations should be presented so that the answers to the individual multiplication steps are on the same line. Children should be encouraged to discuss what is similar and what is different.

- HTO x TO

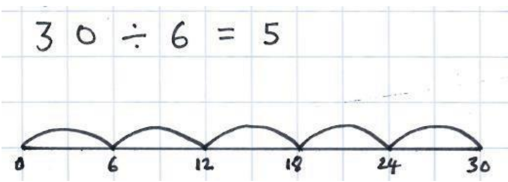




DIVISION

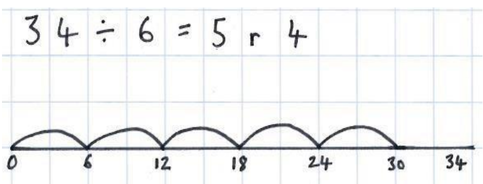
2014 National Curriculum: Division			
Year 3: Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods	**Year 5:** Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context	**Year 6:** divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context	**Year 6:** Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context

1. Grouping



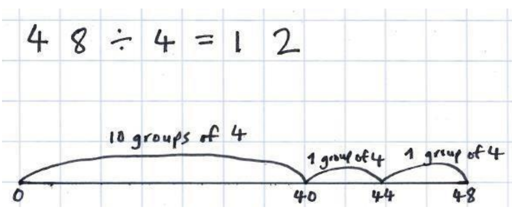
A number line counting up from zero. This representation should be supported by grouping of concrete materials and other pictorial representations. Note: National Curriculum times table requirements should be considered when choosing examples for the early written division methods. National Curriculum requirements are by the end of Y2: x2, x5 and x10. By the end of Y3: x3, x4 and x8. By the end of Y4: up to 12 x 12.

2. Grouping with remainders



Again using a number line counting up from zero. This should also be supported by grouping of concrete materials and other pictorial representations.

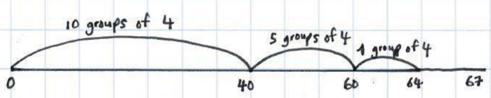
3. Efficient Grouping



Rather than counting individually, children now use groups for efficiency. The number of groups should be recorded above the jump.

4. Efficient Grouping (with remainders)

67 ÷ 4 = 16 r 3



The efficient grouping method now incorporates remainders.

5. Short Division

```
   2 4
4 | 9 ¹6
```

After introduction, short division is extended into dividing up to three digits by one digit. Children should be able to interpret remainders appropriately for the context.

6. TO ÷ O Formal method with chunking

- TO ÷ O (with no remainders)

```
96 ÷ 4 = 24

    2 4
4 | 9 6
  - 4 0 (10 x 4)
    5 6
  - 4 0 (10 x 4)
    1 6
  - 1 6 (4 x 4)
    0 0
```

The chunking method is introduced but only with a single digit divisor. The number of groups should be recorded alongside on the right with the answer written on top of the division bracket

- TO ÷ O (with remainders)

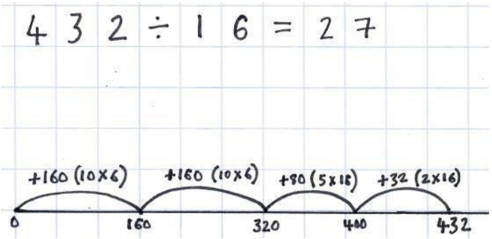
```
43 ÷ 3 =

    1 4 r 1
3 | 4 3
  - 3 0 (10 x 3)
    1 3
  - 1 2 (4 x 3)
    0 1
```

The same layout is then used again but with remainders.

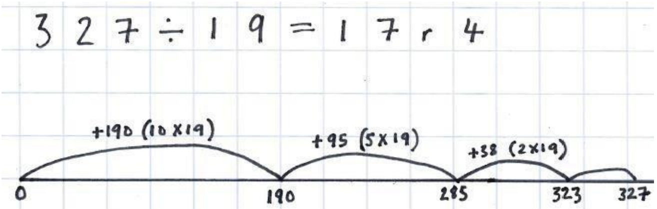
7. Efficient Grouping (÷ 2 digits)

- HTO ÷ TO (without remainders)



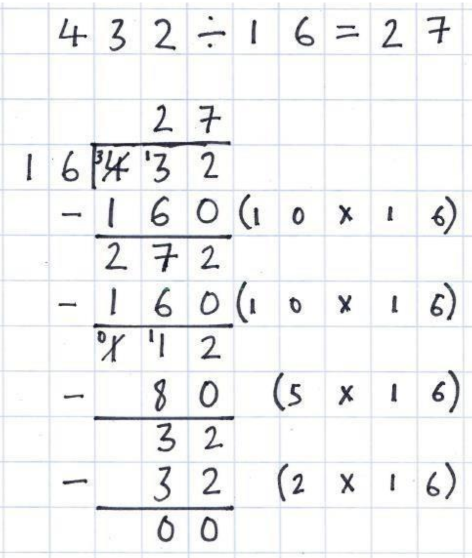
The efficient grouping method is now used again but with a two digit divisor. Again, the number of groups should be recorded above the jump.

- HTO ÷ TO (with remainders)

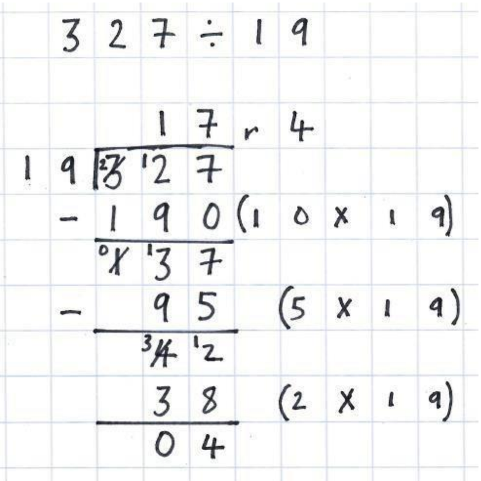


8. Chunking (÷ 2 digits)

- HTO ÷ TO (without remainders)

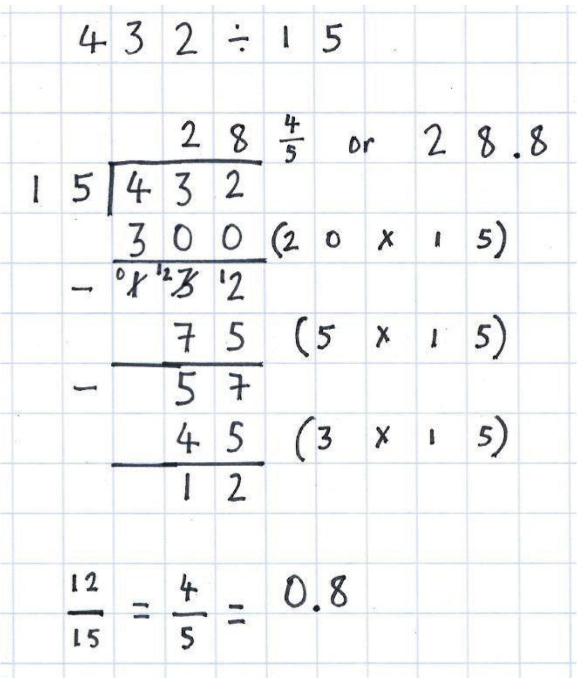


- HTO ÷ TO (with remainders)



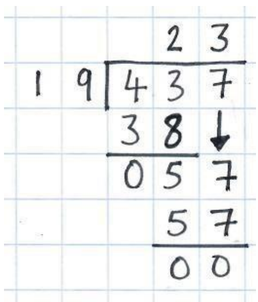
The formal chunking method is reintroduced with a two digit divisor.

- HTO ÷ TO (remainders interpreted as fractions or decimals)



The final stage of chunking is for remainders to be interpreted as fractions, decimals or by rounding as appropriate to the context.

9. Formal Long Division

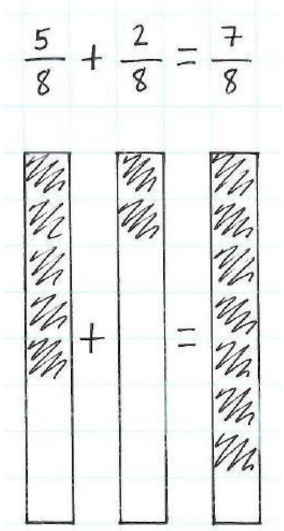


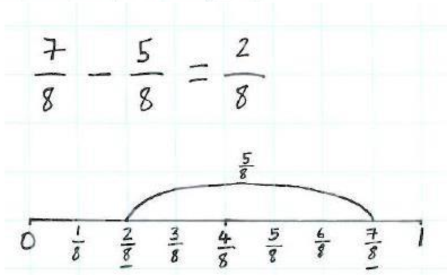
The formal long division method is introduced. Where appropriate, children should be interpreting remainders as whole numbers, fractions, decimals or rounding as required.

FRACTIONS

2014 National Curriculum: Adding & Subtracting Fractions			
Year 3: Add and subtract fractions with the same denominator within one whole	**Year 4:** Add and Subtract fractions where the answer may be an improper fraction	**Year 5:** Add fractions with the same denominators and convert the answer from improper fractions to mixed numbers **Year 5:** Add and subtract fractions where one denominator is a multiple of the other	**Year 6:** Add and subtract fractions with different denominators **Year 6:** Add and subtract a mixed number to a fraction where there are different denominators

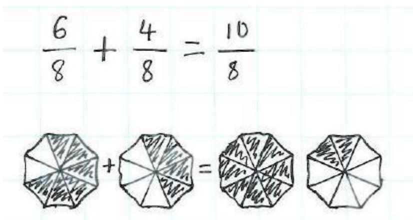
1. Add and Subtract fractions with the same denominator within one whole





Children should practice this with multiple representations before completing the abstract calculation. There is no requirement to go beyond a whole.

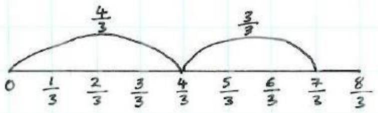
2. Add and Subtract fractions where the answer may be an improper fraction



In Year 4, the expectation moves beyond a whole into improper fractions. There is no requirement to convert to mixed numbers.

3. Add fractions with the same denominators and convert the answer from improper fractions to mixed numbers

$$\frac{4}{3} + \frac{3}{3} = \frac{7}{3}$$



$$7 \div 3 = 2\frac{1}{3}$$

The next step, in Year 5, now requires children to convert their answer from an improper fraction to a mixed number.

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4. Add and subtract fractions where one denominator is a multiple of the other

$$\frac{2}{6} + \frac{1}{3} = \frac{4}{6}$$

$\times 1 \qquad \times 2$

$$\frac{2}{6} + \frac{2}{6} = \frac{4}{6}$$

$$\frac{3}{4} + \frac{4}{8} = \frac{10}{8}$$

$\times 2 \qquad \times 1$

$$\frac{6}{8} + \frac{4}{8} = \frac{10}{8} = 1\frac{2}{8}$$

Children are converting to find the lowest common multiple for the first time – a secure understanding of equivalent fractions is therefore required. In the first instance, examples should be used that remain within a whole before practising with mixed numbers. Children draw upon their knowledge of multiples to find the lowest common multiple rather than multiplying the two denominators.

5. Add and subtract fractions with different denominators

$$\frac{1}{3} + \frac{2}{5} =$$

$\times 5 \qquad \times 3$

$$\frac{5}{15} + \frac{6}{15} = \frac{11}{15}$$

$$\frac{6}{8} - \frac{1}{6}$$

$\times 3 \qquad \times 4$

$$\frac{18}{24} - \frac{4}{24} = \frac{14}{24}$$

At this stage, both common denominators are converted to the lowest common multiple. Children will need to draw upon their times tables knowledge in identifying these. It may be useful to provide additional support through the inclusion of an arrow indicating what the numerator and denominator are being multiplied by when finding the equivalent fraction.

6. Add and subtract a mixed number to a fraction where there are different denominators

$$\frac{5}{15} + 1\frac{1}{3} =$$

$$\frac{5}{15} \xrightarrow{\times 1} \frac{5}{15} \qquad 1\frac{1}{3} \xrightarrow{\times 3} 1\frac{5}{15}$$

$$\frac{5}{15} + 1\frac{5}{15} = 1\frac{10}{15} = 1\frac{2}{3}$$

$$1\frac{2}{5} - \frac{3}{10} =$$

$$1\frac{2}{5} \xrightarrow{\times 2} 1\frac{4}{10} \qquad \frac{3}{10} \xrightarrow{\times 1} \frac{3}{10}$$

$$1\frac{4}{10} - \frac{3}{10} = 1\frac{1}{10}$$

The final stage requires children to again identify the lowest common multiple. A possible misconception here is that children may, in finding the equivalent fraction, multiply the whole number.

2014 National Curriculum: Multiplying Fractions	
Year 5: Multiply proper fractions and mixed numbers by whole numbers	**Year 5:** Multiply simple pairs of proper fractions writing the answer in its simplest form

1. Multiply proper fractions and mixed numbers by whole numbers

$$\frac{2}{3} \times 4 = \frac{2}{3} \times \frac{4}{1} = \frac{8}{3} = 2\frac{2}{3}$$

Proper Fractions: This should be introduced through repeated addition alongside a representation for the majority. With more confident children, you may want to go onto the formal method where they are required to convert the whole into an improper fraction (with 1 as the denominator).

$$2\frac{3}{4} \times 3 = 6\frac{9}{4} = 8\frac{1}{4}$$

$$2 \times 3 = 6$$

$$\frac{3}{4} \times 3 = \frac{9}{4}$$

Mixed Numbers: The whole numbers should be multiplied first before multiplying the proper fraction through repeated addition (as covered in the proper fraction element of this objective).

2. Multiply simple pairs of proper fractions writing the answer in its simplest form

$$\frac{5}{8} \times \frac{6}{7} = \frac{30}{56} = \frac{15}{23}$$

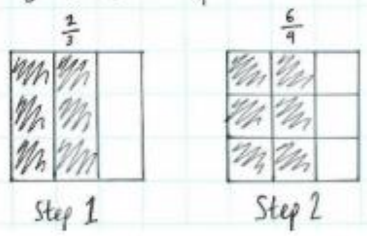
$$\frac{4}{12} \times \frac{3}{4} = \frac{12}{48} = \frac{1}{4}$$

The numerators of both fractions are multiplied together as are the denominators. This should be covered before moving onto the requirement to simplify.

2014 National Curriculum: Dividing Fractions
Year 6: Divide proper fractions by whole numbers (covering when the numerator is and is not a multiple of the whole number)

1. Divide proper fractions by whole numbers (covering when the numerator is and is not a multiple of the whole number).

$$\frac{2}{3} \div 3 = \frac{2}{9}$$



$$\frac{2}{3} \div 3 = \frac{2}{3} \div \frac{3}{1} = \frac{2}{9}$$

$$\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$$

Children should explore the pictorial representation of dividing a fraction. For example, $\frac{1}{2} \div 2$ means children need to split one half into two equal pieces. In the first example, two thirds is divided into three equal parts giving $\frac{6}{9}$. $\frac{6}{9}$ is then divided by 3 to give an answer of $\frac{2}{9}$.

In the abstract method, the whole number is made into an improper fraction before it is changed to a reciprocal of the divisor. The final step is multiplying as explained previously.

Children should be exposed to examples where the numerator is and is not a multiple of the whole number (for example $\frac{12}{15} \div 6$ and $\frac{4}{5} \div 3$).

SUGGESTED VOCABULARY

Addition & Subtraction

Years 3 and 4: add, addition, more, plus, increase, sum, total, altogether, double, near double, how many more to make…? how many more to make…? how many more is… than…? how much more is…? -, subtract, subtraction, take (away), minus, decrease, leave, how many are left/left over? how many fewer is… than…? how much less is…? difference between, half, halve, how many more/fewer is… than…? how much more/less is…? Is equal to, is the same as, tens boundary, hundreds boundary, inverse

Years 5 and 6: add, addition, more, plus, increase, sum, total, altogether, double, near double, how many more to make…? subtract, subtraction, take (away), minus, decrease, leave, how many are left/left over? difference between, half, halve, how many more/fewer is… than…? how much more/less is…? Is equal to, sign, is the same as, tens boundary, hundreds boundary, units boundary, tenths boundary, inverse

Multiplication & Division

Year 3 and 4: lots of, groups of, times, multiply, multiplication, multiplied by, multiple of, product, once, twice, three times… ten times…times as (big, long, wide… and so on), repeated addition, array, row, column, double, halve, share, share equally, one each, two each, three each…group in pairs, threes… tens, equal groups of, divide, division, divided by, divided into, remainder, factor, quotient, divisible by, inverse

Years 5 and 6: lots of, groups of, times, multiply, multiplication, multiplied by, multiple of, product once, twice, three times… ten times…times as (big, long, wide… and so on), repeated addition array, row, column, double, halve, share, share equally, one each, two each, three each…group in pairs, threes… tens, equal groups of, divide, division, divided by, divided into, dividend, divisor, remainder, factor, quotient, divisible by, inverse, fraction